

Searching Algorithms

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Introduction

.Searching for

.a book in library

.Subscriber's telephone number in the telephone directory

.Name in the electoral rolls

Linear Search

- .Sequential search
- .Searched for a key one after the other
- .Ordered and unordered search

Ordered Linear Search

. $L = \{k_1, k_2, k_3, \dots, k_n\}$ such that $k_1 < k_2 < k_3 < \dots < k_n$

.To search key k in the list

.Compare key k with each element so long as $k > k_i$

.If $k = k_i$ then search is done

.Otherwise search is unsuccessful

. $L[0:5] = \{16, 18, 56, 78, 90, 100\}$

.Search 78 and 20

Ordered linear search for $k=78$

.16,18,56,78,90,100

16	18	56	78	90	100
----	----	----	----	----	-----

0 1 2 3 4 5

↑

i

$i < n$ and $k > L[i]$

Ordered linear search for $k=78$

.16,18,56,78,90,100

16	18	56	78	90	100
0	1	2	3	4	5

↑
i

$i < n$ and $k > L[i]$

Ordered linear search for $k=78$

.16,18,56,78,90,100

16	18	56	78	90	100
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$i < n$ and $k > L[i]$

Ordered linear search for k= 78

.16,18,56,78,90,100

16	18	56	78	90	100
0	1	2	3	4	5

↑

i

$K=L[i]$, so key is found

Ordered linear search for $k=20$

.16,18,56,78,90,100

16	18	56	78	90	100
----	----	----	----	----	-----

0 1 2 3 4 5

↑

i

$i < n$ and $k > L[i]$

Ordered linear search for $k=20$

.16,18,56,78,90,100

16	18	56	78	90	100
0	1	2	3	4	5

↑
i

$i < n$ and $k > L[i]$

Ordered linear search for $k=20$

.16,18,56,78,90,100

16	18	56	78	90	100
0	1	2	3	4	5

↑
i

$K < L[i]$, so key not found.

```
Procedure OrderedSearch(L, n, k)
    i=0;
    while ((i<n) and (k>L[i])) do
        i=i+1;
    endwhile
    if (k=L[i]) then {print (“Key found”); return (i); }
    else
        print (“Key not found”);
    end OrderedSearch
```

Unordered Linear Search

. $L = \{k_1, k_1, k_3, \dots, k_n\}$

.To search key k in the list

.Compare key k with each element so long as $k = k_i$

.If $k = k_i$ then search is done

.Otherwise search is unsuccessful

. $L[0:5] = \{23, 14, 98, 45, 67, 53\}$

.Search for 98 and 110

Unordered linear search for k= 98

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 98

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 98

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$K==L[i]$, so key is found.

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
----	----	----	----	----	----

0 1 2 3 4 5

↑

i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5



i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5



i

$i < n$ and $k \neq L[i]$

Unordered linear search for k= 110

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i $i < n$ and $k \neq L[i]$

Unordered linear search for $k=110$

.23,14,98,45,67,53

23	14	98	45	67	53
0	1	2	3	4	5

↑

i

$i > n$, so key not found.

Procedure UnorderedSearch(L, n, k)

 i=0;

 while ((i<n) and (k<>L[i])) do

 i=i+1;

 endwhile

 if (k=L[i]) then {print (“Key found”); return (i); }

 else

 print (“Key not found”);

 end UnorderedSearch

Transpose Sequential Search

.Self Organizing Sequential Search

.Searches a list of data items for a key, checking itself against the data items one at a time in a sequence

.If the key is found, then it is swapped with its predecessor and the search is termed successful.

.Favours faster search when one repeatedly looks for a key.

Search Key	List L before search	Number of element comparisons made during the search	List L after search
90	{34,21,89,45,12,90,76,62}	6	{34,21,89,45, 90,12 ,76,62}
89	{34,21,89,45,90,12,76,62}	3	{34, 89,21 ,45,90,12,76,62}
90	{34,89,21,45,90,12,76,62}	5	{34,89,21, 90,45 ,12,76,62}
21	{34,89,21,90,45,12,76,62}	3	{34, 21,89 ,90,45,12,76,62}
90	{34,21,89,90,45,12,76,62}	4	{34,21, 90,89 ,45,12,76,62}
90	{34,21,90,89,45,12,76,62}	3	{34, 90, 21 ,89,45,12,76,62}

.Procedure TransposeSequentialSearch(L, n, k)

 i=0;

 while ((i<n) and (k<>L[i])) do

 i=i+1;

 endwhile

 if (k=L[i]) then {print (“Key found”);

 swap(L[i], L[i-1]);

 return (i); }

 else

 print (“Key not found”);

 end TransposeSequentialSearch

Interpolation Search

.List must be ordered

.Like dictionary search – we turn sheaves of pages back or forth

.Suppose $L=\{k_1,k_2,k_3,\dots,k_n\}$ and $k_1<k_2<k_3<\dots<k_n$

.When it is known that k lies between K_{low} and k_{high} then the next element to be probed is $(k-low)/(k_{high}-k_{low})$

Implementation

.mid = $i + ((j-i) \cdot (k - k_i) / (k_j - k_i))$

.Comparison cases

.If ($k = k_{mid}$) the the search is done.

.If ($k < k_{mid}$) then continue the search in the sublist $\{k_i, k_{i+1}, k_{i+2}, \dots, k_{mid-1}\}$

.If ($k > k_{mid}$) then continue the search in the sublist $\{k_{mid+1}, k_{mid+2}, \dots, k_j\}$

Example

.L={24,56,67,78,79,89,90,95,99}

.Search for the keys 67 and 45

K	i	j	found	mid	< K=L[mid] >
67	1 1	9 4	False False True	$1 + ((9-1) * (67-24) / (99-24)) = 5.58 = 5$ 3 Key found	67 < (L[5] = 79) 67 = (L[3] = 67)
45					

K	i	j	found	mid	< K=L[mid] >
67	1	9	False	$1 + ((9-1) * (67-24) / (99-24)) = 5.58 = 5$	$67 < (L[5] = 79)$
	1	4	False	3	$67 = (L[3] = 67)$
			True	Key found	
45	1	9	False	$1 + ((9-1) * (45-24) / (99-24)) = 3.24 = 3$	$45 < (L[3] = 67)$
	1	2	False	1	$45 > (L[1] = 24)$
	2	2	False	2	$45 < (L[2] = 56)$
	2	1	False	Key not found	

Algorithm

.Procedure Interpolation_Search(L, n, k)

i=1; j=n;

If ($k < L[i]$) or ($k > L[j]$) then {print (“Key not found”) ;
exit();}

while (($i \leq j$) and (found=false)) do

mid= $i + (j - i) \cdot (k - L[i]) / (L[j] - L[i])$;

case

: $k = L[\text{mid}]$: {found=true; print (“Key found”);

: $k < L[\text{mid}]$: $j = \text{mid} - 1$;

: $k > L[\text{mid}]$: $i = \text{mid} + 1$;

endcase

if (found=false) then print(“Key not found”);

end Interpolation_Search

Binary Search

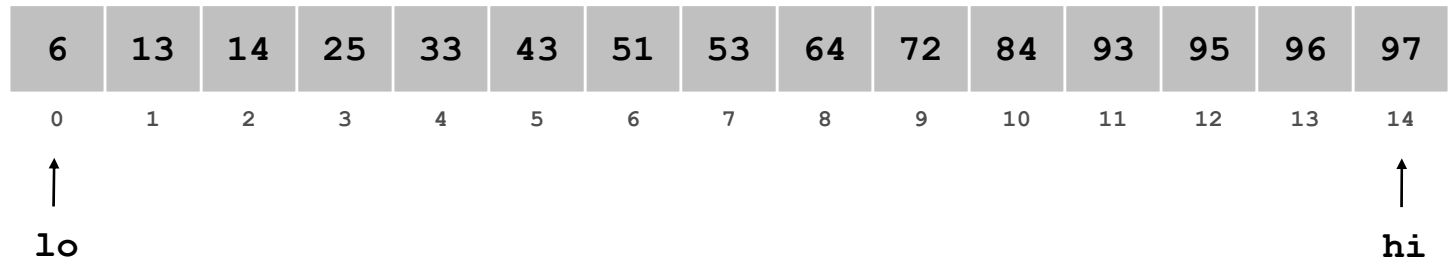
.Locates a target value in a *sorted* array/list by successively eliminating half of the array from consideration.

.Binary search. Given value and sorted array $a[]$, find index i such that $a[i] = \text{value}$, or report that no such index exists.

.Algorithm maintains $a[\text{low}] \leq \text{value} \leq a[\text{high}]$.

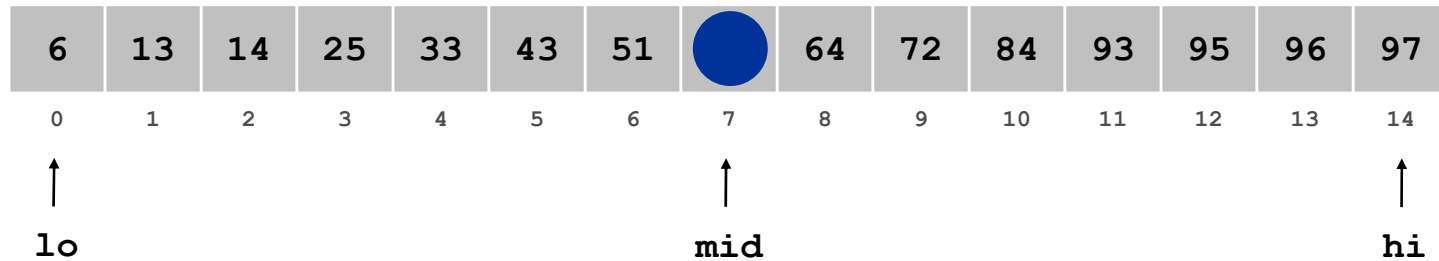
Binary Search

Ex. Binary search for 33.



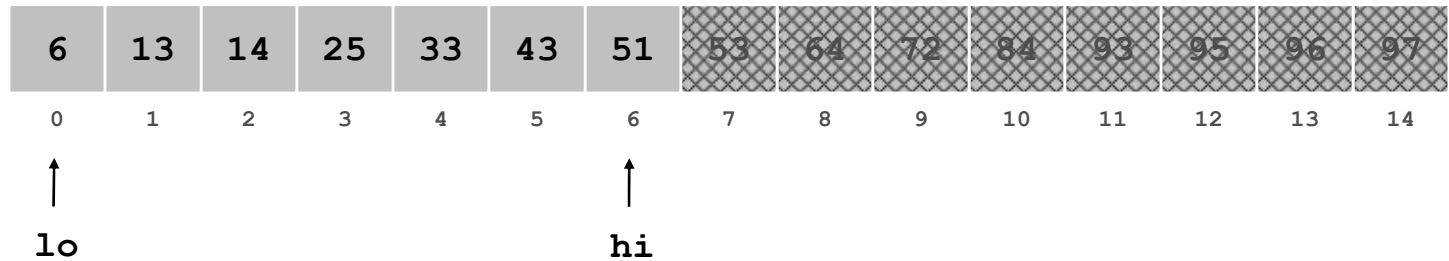
Binary Search

Ex. Binary search for 33.



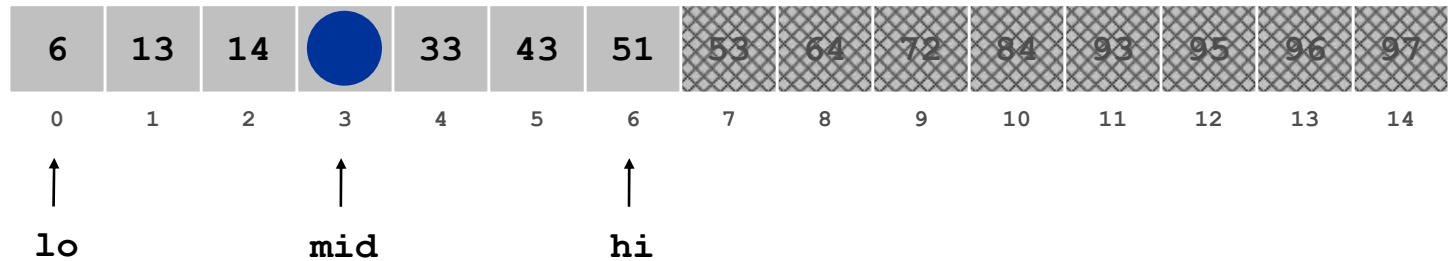
Binary Search

Ex. Binary search for 33.



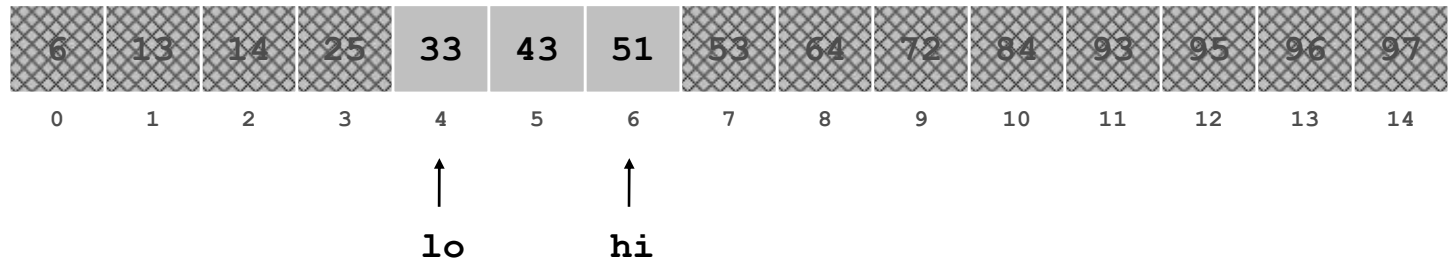
Binary Search

Ex. Binary search for 33.



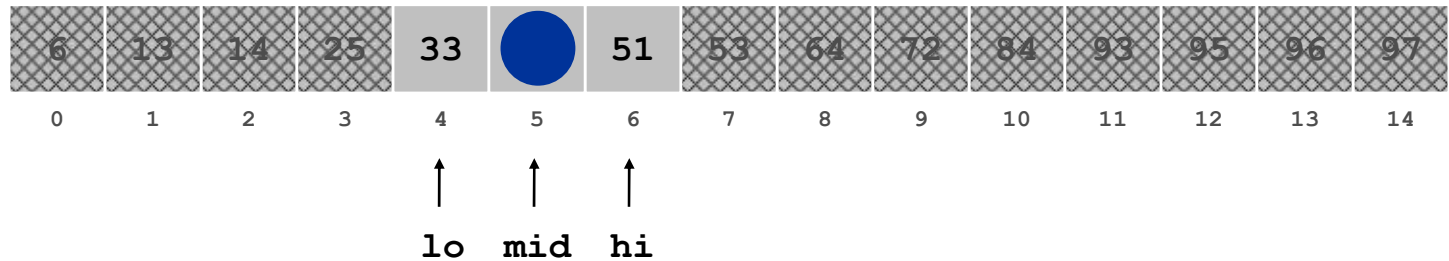
Binary Search

Ex. Binary search for 33.



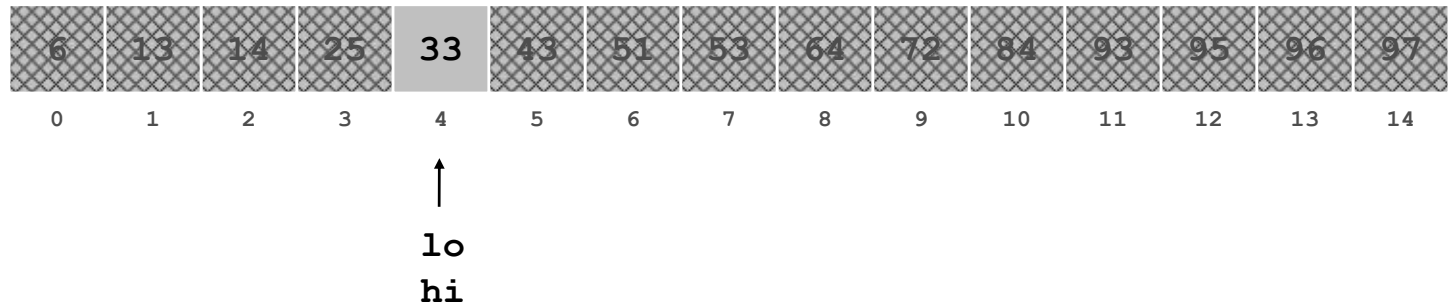
Binary Search

Ex. Binary search for 33.



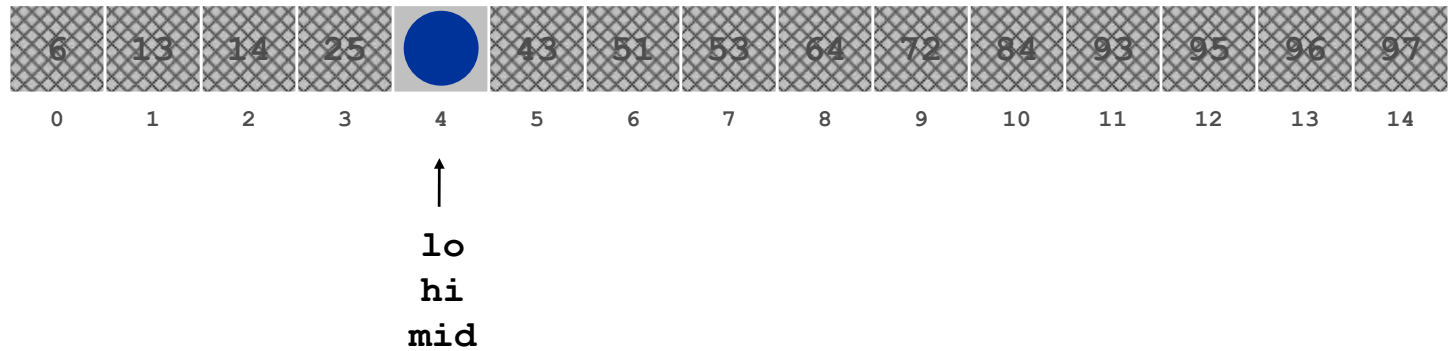
Binary Search

Ex. Binary search for 33.



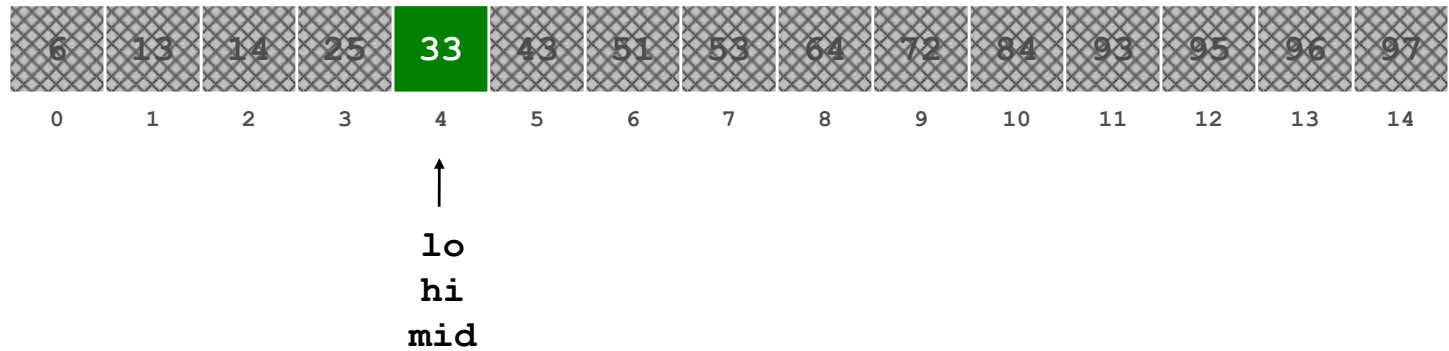
Binary Search

Ex. Binary search for 33.



Binary Search

Ex. Binary search for 33.



```

.Procedure Binary_Search(L, low, high, k)
if (low>high) then { binary_search=0;
                    print("Key not found"); exit(); }
else {
    mid=(low+high)/2;
    case
    : k = L[mid] : {print("Key found");
                  binary_search=mid; return L[mid]; }
    : k < L[mid] : binary_search =
                  Binary_Search(L, low, mid-1, k);
    : k > L[mid] : binary_search =
                  Binary_Search(L, mid+1, high, k);
    endcase
}
end Binary_Search

```

Fibonacci Search

.Works only on ordered list

.Fibonacci sequence $\{0, 1, 1, 2, 3, 5, 8, 13, 21, \dots\}$

. $F_0 = 0$

. $F_1 = 1$

. $F_i = F_{i-1} + F_{i-2}$

.Next element of comparison as dictated by Fibonacci number sequence

.Number of elements in the list is one less than a Fibonacci number i.e. $n = F_k - 1$

Example

Search Key K	$<$ $K = L[p]$ $>$	T	p	q	r	Remarks
434	$k > L[13] = 221$ $k > L[13] = 221$ $k > L[18] = 401$ $k > L[20] = 536$ $k > L[19] = 434$	1	13 13 18 20 19	8 8 3 1 1	5 5 2 1 0	$n=20$ $m=0$ since $F8 + 0 = n+1$ Since $k > L[p]$, $p=p+m$ Key is found
66	$K < L[13] = 221$ $k > L[8] = 55$ $k < L[11] = 94$ $k < L[10] = 84$ $k < L[9] = 81$	8 2 1	13 8 11 10 9	8 5 2 1 1	5 3 1 1 0	$n=20$ $m=0$ Since $r=0$, p is set to 0. Key is not found.

Algorithm

Procedure Fibonacci_Search(L, n, k)

Obtain the largest fibonacci number F_k closest to $n=n+1$;

$p = F_{k-1}$;

$q = F_{k-2}$;

$r = F_{k-3}$;

$m = (n+1)-(p+q)$;

if ($k > L[p]$) then $p = p+m$;

found = false;

while (($p <> 0$) and (not found)) do

 case

 : $k = L[p]$: { print("Key found"); found = true; }

 : $k < L[p]$: if ($r=0$) then $p=0$;
 else { $p=p-r$; $t=q$; $q=r$; $r=t-r$; }

 : $k > L[p]$: if ($q=1$) then $p=0$
 else { $p=p+r$; $q=q-r$; $r=r-q$; }

 endcase

endwhile

if (found=false)then print("Key not found");

end Fibonacci_Search

Other Search Techniques

- .Internal Searching

- .External Searching

 - .Tree Search

 - .Graph Search

 - .Indexed Sequential Search